CANONICAL REPRESENTATION AND FIGURE OF MERIT OF CIRCULATOR

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Abstract

Proposing a canonical representation, valid for analysis and synthesis, of circulator, the paper also attempts to prove the circulator's figure of merit to be invariant under an arbitrary lossless reciprocal and cyclic-symmetry imbedding.

I. Introduction

In a microwave communication system, circulator is an important device which can separate an incident signal and a reflected one, and so it is utilized in a refection type of amplifier and phase-modulator, a switch and even as a buffer when one port of it is terminated to a dummy load.

A lossless reciprocal compensating network is often employed in order to transform an "actual" circulator which is almost imperfect, into an "ideal" one which has an optimum performance, i.e., perfect matching and perfect isolation.

Conventionally, the compensating network for the circulator is composed of three identical 2-port reactance networks shown in Fig.1(a). However, we will propose a more general compensating network which is a 6-port network and is shown in Fig.1(b), and study the ability of the compensating network. From these investigations, an invariant number proper to the circulator will be found to be an suitable figure of merit for the circulator performance. Furtheremore, the canonical form and the general form of the circulator, valid for analysis and synthesis, can be obtained.

In addition, we will investigate the ability of the conventional compensating network and the degradation by the losses in the compensationg network.

In this paper, discussions are concentrated on the performance at a specified frequency.

II. Lossless reciprocal transformation with preserving a cyclic-symmetry

At first, we will consider the properties of the compensating network. Let us assume a tandem connection of two 6-port reactance networks with preserving a cyclic-symmetry shown in Fig.2. A network preserving a cyclic-symmentry has a following scattering matrix.

$$\Sigma_{ij} \cdot R = R \cdot \Sigma_{ij} \qquad (i,j=1,2) \tag{1}$$

where,

$$R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \tag{2}$$

and Σ_{ij} are the 3x3 submatrices of Σ .

$$\Sigma = \begin{bmatrix} \Sigma_{11}, & \Sigma_{12} \\ \Sigma_{21}, & \Sigma_{22} \end{bmatrix} \tag{3}$$

On the other hand, the losslessness and the reciprocity of the network means that

$$\bar{\Sigma}^t \cdot \Sigma = E \tag{4}$$

$$\Sigma^t = \Sigma \tag{5}$$

, respectively. The bar and t denotes complex conjugate and transposition of the matrix, respectively. E is a unit matrix.

It will be shown after algebraic manipulation that the resulting 6-port network has the same characters as the individual networks, i.e., the losslessness, the reciprocity and the cyclic-symmetry. Moreover, the "unit" network (not unit matrix) and the "inverse" network (not inverse matrix) with respect to the operation of "tandem connection", which are lossless and reciprocal and preserve a cyclic-symmetry, can always exist uniquely. These explicit expressions are given as follows;

$$\Sigma_E = \begin{bmatrix} 0 & E \\ E & 0 \end{bmatrix} \tag{6}$$

$$\Sigma^{-1} = \begin{bmatrix} \Sigma_{12}^{-1}(\Sigma_{22}\Sigma_{12}^{-1} - \Sigma_{21} \cdot \Sigma_{11}^{-1})^{-1} & \Sigma_{22}^{-1}(\Sigma_{12}\Sigma_{22}^{-1} - \Sigma_{11}\Sigma_{21}^{-1})^{-1} \\ \Sigma_{11}^{-1}(\Sigma_{21}\Sigma_{11}^{-1} - \Sigma_{22}\Sigma_{12}^{-1})^{-1} & \Sigma_{21}^{-1}(\Sigma_{11}\Sigma_{21}^{-1} - \Sigma_{12}\Sigma_{22}^{-1})^{-1} \end{bmatrix} \tag{7}$$

Therefore, the set of 6-port compensating networks can be said to be the "group" with respect to tandem connection.

III. Invariant of cyclic-symmetry 3-port network under lossless reciprocal and cyclic-symmetry transformation

Because of the cyclic-symmetry in the given circulator and the compensated circulator, these 3-port networks can be decomposed into three "eigen" 1-port networks. (See Fig.3) Therefore, the transformation $\Sigma : S \to \tilde{S}$ can be also decomposed into three transformations $\Sigma^k : S^k \to \tilde{S}^k (k=0,1,2)$, where S and $\tilde{S}$ means the scattering matrix of the given circulator and the compensated circulator, respectively, and the quantities with superscript 0,1 and 2, means the one of "co-phase", "clockwise" and "anti-clockwise" eigen vector, respectively.

The reciprocity of Σ restricts the eigen values of submatrices $\Sigma_{11}, \Sigma_{12}, \Sigma_{21}$ and Σ_{22} which are denoted by $\Sigma_{11}^k, \Sigma_{12}^k, \Sigma_{21}^k$ and Σ_{22}^k, respectively.

$$\Sigma_{11}^1 = \Sigma_{11}^2, \quad \Sigma_{22}^1 = \Sigma_{22}^2$$

$$\Sigma_{12}^0 = \Sigma_{21}^0, \quad \Sigma_{12}^1 = \Sigma_{21}^2, \quad \Sigma_{12}^2 = \Sigma_{21}^1 \tag{8}$$

As a result, the transformation of clockwise eigen excitation is shown to be the same as that of anti-clockwise eigen excitation.

Furtheremore, the losslessness of Σ leads to a conclusion that three individual transformations Σ^k (k=0,1,2) are also lossless.

From these facts, the reflection coefficients of the given circulator for clockwise and anti-clockwise excitation, S^1 and S^2 , respectively, must be transformed by the identical lossless transformation;

$$S^k \to \tilde{S}^k = \Sigma_{11}^1 + \Sigma_{12}^1 \cdot \Sigma_{12}^2 \cdot S^k / (1 - \Sigma_{22}^1 S^k) \quad (k=1,2) \tag{9}$$

Kawakami[2] and Kurokawa et. al.[3] show that the following quantity is invariant under these transformations.

$$\frac{|S^1 - S^2|}{|1 - S^1 \cdot \bar{S}^2|} = \frac{|\tilde{S}^1 - \tilde{S}^2|}{|1 - \tilde{S}^1 \cdot \bar{\tilde{S}}^2|} \quad (= m) \tag{10}$$

On the other hand, the transformation of co-phase excitation is independent of the other two transformations. Thus, m is the only invariant of the circulator.

IV. Canonical form and general form of circulator

Perfect circulator action which means perfect matching and perfect isolation, will be obtained if the following two conditions among three eigen reflection coefficients S^k (k=0,1,2) are satisfied.

$$S^2 = S^1 \omega \qquad S^1 = S^0 \cdot \omega \tag{11}$$

where, $\omega = \exp(j2\pi/3)$

In such a case, the scattering matrix of 3-port network is of the form given by (12).

$$S = \begin{bmatrix} 0 & S^0 & 0 \\ 0 & 0 & S^0 \\ S^0 & 0 & 0 \end{bmatrix} \tag{12}$$

Then, the question whether we can always transform an imperfect circulator into a perfect one by a suitable compensating network or not, will be rised naturally. The answer to this question is given by Theorem 1.

[Theorem 1] we can always transform an imperfect circulator into a perfect one by a suitable compensating 6-port network and its transmission coefficient is determined by only the given circulator but does not depend on the compensating network.

(Proof) The restriction on the three transformations Σ^k (k=0,1,2) is only Eq.(10). So, let us assume that the condition (11) can be satisfied. Eq.(11) is substituted into Eq.(10).

$$m^2 = \frac{|S^1 - S^2|^2}{|1 - S^1 \cdot \bar{S}^2|^2} = \frac{|\tilde{S}^1 - \tilde{S}^2|^2}{|1 - \tilde{S}^1 \cdot \bar{\tilde{S}}^2|^2} = \frac{3|\tilde{S}^1|^2}{1 + |\tilde{S}^1|^2 + |\tilde{S}^1|^4} \tag{13}$$

From (13), the solution to $|\tilde{S}^1|^2$ equal to or less than unity, is always gained uniquely.

$$|\tilde{S}^1|^2 = \frac{3/m^2 - 1}{2} - \sqrt{\left(\frac{3/m^2 - 1}{2}\right)^2 - 1} \quad (= \alpha^2) \tag{14}$$

We can, therefore, conclude that an imperfect circulator can be always transformed into a perfect one by a suitable compensating network and α given by (14) is also invariant, because m is invariant and α is a monotonic increasing function of m. In other words, the transmission coefficient is determined by only the given circulator.

Such a perfect circulator will be referred to as a "canonical form" of circulator. Moreover, the following corollary is easily proved, because the inverse of any compensating network always exists.

[Corollary] An actual imperfect circulator can be represented by a parastic reactance part and a perfect circulator part shown in Fig.4.

We shall call this representation a "general form" of circulator. Such a representation is valid for analysis and synthesis of the circulator.

V. Figure of merit associated with circulator

Next, we will expose that α is a suitable figure of merit of circulator.

[Lemma] n($\geq$2) circulators with equal figure of merit α, are imbedded into 3(n+1)-port reactance network preserving a cyclic-symmetry. The figure of merit of the resulting circulator is denoted by α_t.

1) For arbitrary choice of the transforming network, α_t can not exceed α.
2) By a proper choice of the transforming network, α_t can assume arbitrary value in the interval $[0, \alpha]$.

(Proof) We will here employ an impedance matrix instead of a scattering matrix for the convenience.

The transforming (n+1)-port networks of clockwise and anti-clockwise excitation, $[z^1]$ and $[z^2]$ are lossless but not reciprocal in general, however, $[z^1]$ is a transposition of $[z^2]$.

$$[z^1] = [z^2]^t \tag{15}$$

The losslessness is reflected in the form;

$$[z^1] = \begin{bmatrix} jx_0 & a \\ -\bar{a}^t & \hat{z} \end{bmatrix} \tag{16}$$

where, $[\hat{z}] + [\bar{\hat{z}}]^t = 0$ (17)

Hence, the transformed eigen impedances of clockwise and anti-clockwise excitation are given as follows;

$$\tilde{z}_1 = jx_0 - a \cdot (\hat{z} + z_1 E)^{-1} \cdot \bar{a}^t \tag{17}$$

$$\tilde{z}_2 = jx_0 - a \cdot (\hat{z} + z_2 E)^{-1} \cdot \bar{a}^t \tag{18}$$

Because $[\hat{z}]$ is a skew Hermite matrix, all its eigen values are pure imaginary numbers.

$$[\hat{z}] = U \cdot \begin{bmatrix} jx_1 & & 0 \\ & \ddots & \\ 0 & & jx_n \end{bmatrix} \bar{U}^t \tag{19}$$

where, [u] is a unitary matrix. From (17) and (18),

$$\tilde{z}_1 - \tilde{z}_2 = (z_1 - z_2)\, b \cdot \begin{bmatrix} 1/(jx_1+z_2)(jx_1+z_1) & & 0 \\ & \ddots & \\ 0 & & 1/(jx_n+z_2)(jx_n+z_1) \end{bmatrix} \cdot \bar{b}^t \tag{20}$$

where, $b = aU$, and,

$$\tilde{z}_1 + \bar{\tilde{z}}_2 = (z_1 + \bar{z}_2) \cdot b \begin{bmatrix} 1/(jx_1 - \bar{z}_2)(jx_1 + z_1) & & 0 \\ & \ddots & \\ 0 & & 1/(jx_n - \bar{z}_2)(jx_n + z_1) \end{bmatrix} \bar{b}^t \tag{21}$$

Therefore, the invariant of the synthesized circulator can be calculated.

$$m_t = \frac{|\tilde{z}_1 - \tilde{z}_2|}{|\tilde{z}_1 + \bar{\tilde{z}}_2|} = \frac{|z_1 - z_2|}{|z_1 + \bar{z}_2|} \frac{\left|\sum_{i=1}^{n} |b_i|^2/(jx_i + z_2)(jx_i + z_1)\right|}{\left|\sum_{i=1}^{n} |b_i|^2/(jx_i - \bar{z}_2)(jx_i + z_1)\right|} \tag{22}$$

Considering $z_1 = \bar{z}_2$ and Schwartz's inequality,

$$m_t \leq m \tag{23}$$

For α is a monotonic increasing function of m,

$$\alpha_t \leq \alpha \tag{24}$$

Moreover, it is easily shown that α_t can assume arbitrary number in the interval $[0, \alpha]$, by choosing a suitable transforming network.

[Corollary] By imbedding two circulator both figures of merit α into 9-port lossless reciprocal network preserving a cyclic-symmetry, it is always possible to synthesize a circulator with figure of merit α' by a proper choice of transforming 9-port, where $0 \leq \alpha' \leq \alpha$

Using the above lemma and its corollary, we may easily derive the following theorem.

[Theorem 2] If we imbed n circulators with figure of merit $\alpha_1, \alpha_2, \ldots, \alpha_n$ into an 3(n+1)-port reactance network to synthesize an circulator, its figure of merit α_t satisfies the following inequality.

$$\alpha_t \leqq \mathrm{Max}(\alpha_1, \alpha_2, \cdots, \alpha_n) \tag{25}$$

(Proof) Let Max$(\alpha_1, \ldots, \alpha_n)$ be denoted by α. We can synthesize n circulators $\alpha_1, \ldots, \alpha_n$ each using two circulators with figure of merit α, 2n circulators in all. Therefore, the resultant circulator mentioned in Theorem 2 can be synthesized using 2n circulators with figures of merit α. From the above lemma, α_t cannot exceed α.

From these theorems, the meaning of figure of merit will be understood more clealy. In a sense, a group of circulators of poor figure of merit can not substitute for an circulator of high figure of merit.

VI. Conventional compensating network

An ability of a conventional compensating network will be discussed here. The conventional one composed of three 2-port reactance network corresponds to a special case of the 6-port one proposed here. We will discuss the case where the given circulator is lossless, firstly, and then the case where the circulator is lossy.

For lossless circulator

Three eigen reflection coefficient are transformed by the compensating network.

$$\tilde{S}^K = \Sigma_{11} + (\Sigma_{12})^2 \cdot S^K/(1 - \Sigma_{22} S^K) \tag{26}$$

Therefore, the scattering matrix of the compensated circulator has the following matrix elements.

$$\begin{aligned}
\tilde{S}_{11} &= \Sigma_{11} + (\Sigma_{12})^2 \{S^0/(1-\Sigma_{22}S^0) + S^1/(1-\Sigma_{22}S^1) + S^2/(1-\Sigma_{22}S^2)\}/3 \\
\tilde{S}_{12} &= (\Sigma_{12})^2 \{S_0/(1-\Sigma_{22}S^0) + \omega^2 S^1/(1-\Sigma_{22}S^1) + \omega S^2/(1-\Sigma_{22}S^2)\}/3 \\
\tilde{S}_{13} &= (\Sigma_{12})^2 \{S_0/(1-\Sigma_{22}S^0) + \omega S^1/(1-\Sigma_{22}S^1) + \omega^2 S^2/(1-\Sigma_{22}S^2)\}/3
\end{aligned} \tag{27}$$

If the condition $\tilde{S}_{13} = 0$ (perfect isolation) is satisfied, Σ_{22} will be solved by (27).

$$\Sigma_{22} = -\frac{S^0 + \omega S^1 + \omega^2 S^2}{S^0 S^1 \omega^2 + S^0 \cdot S^2 \omega + S^1 S^2} \tag{28}$$

On the other hand, the condition, $\tilde{S}_{12} = 0$ reduces to

$$\Sigma_{22} = -\frac{S^0 + \omega^2 S^1 + \omega S^2}{S^0 S^1 \omega + S^0 S^2 \omega^2 + S^1 S^2} \tag{29}$$

Once $\tilde{S}_{13} = 0$ ($\tilde{S}_{12} = 0$) is established, perfect matching ($\tilde{S}_{11} = 0$) will be guaranteed, because of the losslessness of the compensated network.

It can be verified that Σ_{22} satisfying (28) or (29) has an amplitude less than unity for arbitrary angles of S^0, S^1, and S^2, only except the degenerating cases. So, we can say that a lossless nonreciprocal 3-port network can be transformed into an ideal circulator by a suitable conventional compensating network. The above conclusion, however, cannot be obtained for a lossy 3-port network.

For lossy circulator

The transformation given by (26) will map the region interior to the unit circle centered at the origin of the complex plane into the region interior to the unit circle in the other complex plane. In order to realize a perfect circulator action, three eigen reflection coefficients must have the same amplitude and the differences of their phase angles must be equal to 120°. Arbitrary three points in the unit circle can not always be transformed into the points which are placed to realize a perfect circulator action, because the circle passing through the arbitrary three points will not always lie in the unit circle. Therefore, it can be concluded that a loss nonreciprocal 3-port network can not always be transformed into a perfect circulator by a conventional compensating network. On the other hand, a general 6-port compensating network can always transform a lossy 3-port network into a perfect circulator.

VII. Effect of loss in the compensating network

Here we will discuss the effect of possible loss in the compensating network. The loss maybe distributes in the compensating network, but it can be expressed by unit resistors, concisely, i.e., a lossy 6-port network is equivalent to a lossless 12-port network of which six ports are terminated by six unit resistors.[4]

A network composed of three unit resistors can be thought as a circulator with figure of merit $\alpha = 0$, so that it is concluded by Theorem-2 that a lossy compensating network may diminish the figure of merit of the circulator to be compensated. Therefore, there is no merit in employing a lossy compensating network.

VIII. Conclusion

The conclusions are summarized here.

1) A 6-port compensating network for circulator is proposed
2) An invariant of a cyclic-symmetry 3-port network under lossless reciprocal and cyclic-symmetry imbedding is found by means of eigen vectod excitation.
3) The invariant can be employed as a figure of merit of circulator.
4) Canonical and general form of circulator, varid for analysis and synthesis, is provided.
5) An effect of loss in the compensating network is discussed.

References

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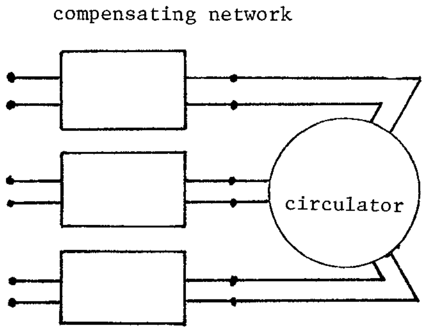
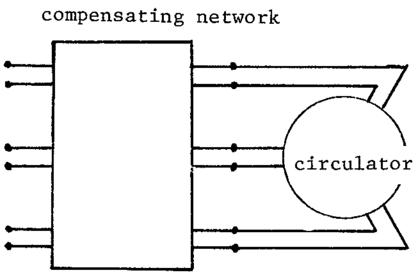


(a) conventional type.



(b) general type.

Fig. 1 A compensating network for circulator.

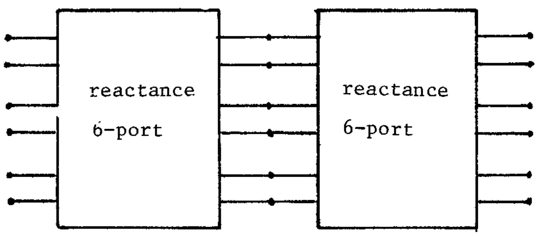


Fig.2 A tandem connection of two reactance 6-port network.

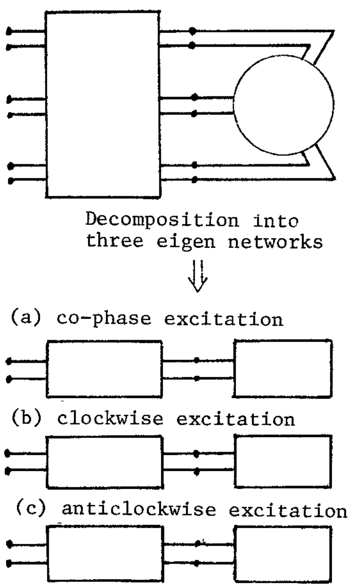


Fig.3 A Fig.3 Decomposition into three "eigen" networks.

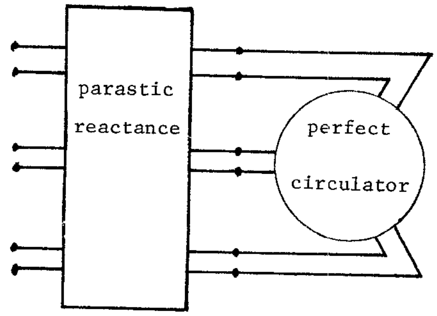


Fig.4 A general representation of circulator.

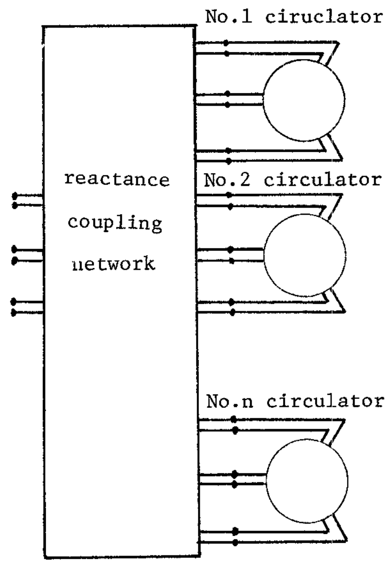


Fig.5 Synthesis of circulator.

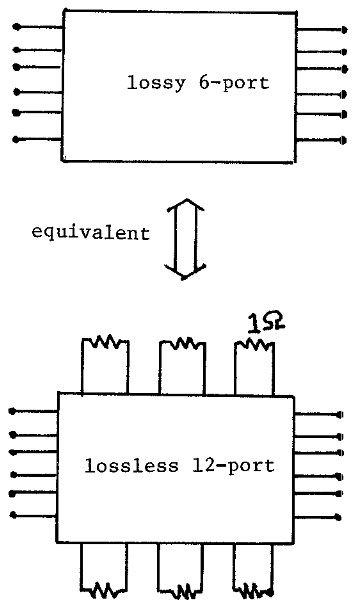


Fig.6 A lossy compensating network.